INCREASING THE ACCURACY OF CALCULATION OF THE VELOCITY FIELD IN A ROTATING ELECTROLYSER WITH AXIAL ELECTROLYTE INLET

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A higher precision in the calculation of the velocity field has been achieved by determining functions of the fifth order in the expansions for velocities. Since the solution was carried out in a new way and quite generally, it turned out that functions of higher orders could also be found by the proposed method if necessary. The assumption that the functions of higher order may influence the individual velocity courses, especially in the inlet region, was substantiated.

A rotating electrolyser with an axial electrolyte inlet[1,2], the velocity field of which will be calculated below, consists of two disc-shaped electrodes rotating at the same angular velocity around a common vertical axis. An electrolyte is fed into the space between them through the axis. Interaction between the radial flow and rotation of the discs gives rise to a strong current at each electrode resulting in a hydromechanical separation of the anolyte and catholyte even when the interelectrode distance is as small[3] as 0·8 mm. Such a small distance is of great advantage in the case of low electrolyte conductivity. A theory of this system was first derived by Kreith and Peube[4,5], who also carried out the first calculation of the velocity field by solving the Navier–Stokes equation for the simple case $\alpha = 1$. Later, Kreith[6] published the solution for various values of the Taylor criterion α ranging from 0 to π and it turned out that the radial velocity in the central plane could be equal to zero. Jansson, Marshall and Rizzo[2] considered the case $\alpha > \pi$ and deduced that the maximum radial velocity shifts towards the discs with increasing α, whereas the velocity should be equal to zero in the plane of symmetry. Measurements[2] by the Doppler anemometer with a laser source substantiated the existence of a rapid streaming at the wall, but strong back streams (depending on the radius) were found, in addition, in the central plane region, which could not be derived from the theory involving functions of the first and third order.

Further experimental work[3,7] showed that the rotation of the discs causes sucking of the liquid between them, the volume rate of flow being surprisingly high, depending on the angular velocity but not on the interelectrode distance. To characterize the mass transfer to the rotating electrodes, the rotation Reynolds number was proposed[3].

The present work deals with the calculation of the velocity field between the rotating discs. Up to now, relations with functions of the first and third order were obtained from the solution of the Navier–Stokes equation, however Jansson[8] suggested that under certain conditions higher-order terms are required to obtain a realistic description of the system considered. We succeeded in deriving implicitly functions of the fifth order and thus obtained a more accurate solution as shown below.

THEORETICAL

The system considered is shown schematicaly in Fig. 1 together with a cylindrical coordinate system. Both discs have the same angular velocity, both the density and the kinematic viscosity of the liquid are constant. The velocity field is described by the Navier-Stokes equation, which gives for the velocity components in the dimensionless form

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \left(\frac{\alpha}{Re}\right)^2 \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{\alpha}{Re}\left[\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + \frac{\partial^2 u}{\partial z^2}\right], \qquad (1)$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{\alpha}{Re}\left[\frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right) + \frac{\partial^2 v}{\partial z^2}\right], \qquad (2)$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\alpha}{Re}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right]. \qquad (3)$$

We assume[8] that the solution of the above system of equations can be written – with regard to the continuity equation – in the form

$$u = \frac{\alpha}{Re}\sum_{i=-1}^{n}\frac{f_i'(z)}{r^i} = \frac{\alpha}{Re}\left[r f_{-1}'(z) + f_0'(z) + r^{-1} f_1'(z) + \ldots\right], \qquad (4)$$

$$w = \frac{\alpha}{Re}\sum_{i=-1}^{n}(i-1)\frac{f_i(z)}{r^{i+1}} = \frac{\alpha}{Re}\left[-2f_{-1}(z) - r^{-1} f_0(z) + \ldots\right], \qquad (5)$$

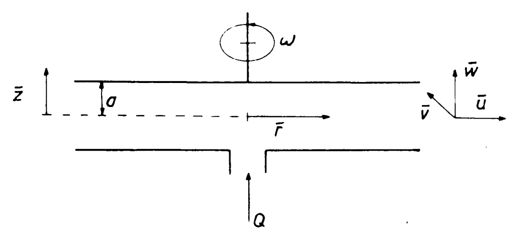

FIG. 1

Scheme of rotating electrolyser with axial electrolyte inlet. $P(0,0,0)$ origin of coordinates, $\bar{u}$ radial velocity, $\bar{w}$ normal velocity, $\bar{v}$ tangential velocity

$$v = \sum_{i=-1}^{n} \frac{g_i(z)}{r^i} = rg_{-1}(z) + g_0(z) + r^{-1} g_1(z) + \ldots, \qquad (6)$$

$$p = \left(\frac{\alpha}{Re}\right)^2 \left(\sum_{i=-1}^{n} \frac{h_{i-1}(z)}{r^{i-1}} + h(z) \ln r\right). \qquad (7)$$

On analysing this system, we arrive at conditions that must be satisfied by the functions f, g, and h. Owing to symmetry with respect to the vertical rotation axis and since we assume also symmetry with respect to the horizontal axis, we have for even n values

$$f_0(z) = f_2(z) = \ldots f_n(z) = 0,$$

$$f'_0(z) = f'_2(z) = \ldots f'_n(z) = 0,$$

$$g_0(z) = g_2(z) = \ldots g_n(z) = 0,$$

$$h_{-1}(z) = h_1(z) = \ldots h_{n-1}(z) = 0.$$

Further, for odd n values ($n = -1, 1, 3 \ldots$) $f_n(z)$ is an odd function, $g_n(z)$ and $h_{n-1}(z)$ are even functions. The boundary conditions are

$$f_n(\pm\alpha) = 0 \quad \text{for} \quad n = -1, 0, 2, 3, 4, \ldots; \quad f_1(\alpha) = \alpha Re,$$

$$f'_n(\pm\alpha) = 0 \quad \text{for} \quad n = -1, 0, 1, 2, \ldots$$

$$g_n(\pm\alpha) = 0 \quad \text{for} \quad n = 0, 1, 2, \ldots; \quad g_{-1}(\pm\alpha) = 1.$$

By introducing these conditions, the expansions for the velocities and static pressure (*4*)–(*7*) are considerably simplified. Then, we introduce them into equations (*1*)–(*3*) and rearrange the resulting equations by putting together the terms with equal powers of r. Thus, we obtain

$$\sum_{i=1}^{(n+3)/2} a_{1i} r^{-2i-3} = a_{11} r + a_{12} r^{-1} + a_{13} r^{-3} + \ldots = 0,$$

$$\sum_{i=1}^{(n+3)/2} a_{2i} r^{-2i+3} = a_{21} r + a_{22} r^{-1} + a_{23} r^{-3} + \ldots = 0, \qquad (8)$$

$$a_{31} r^2 + a_{32} \ln r + \sum_{i=3}^{(n+3)/2} a_{3i} r^{-2i+4} = 0.$$

The unknowns a_{ij} in a given column are polynomials involving functions $f(z)$, $g(z)$, and $h(z)$ at most of the $(2j - 1)$-th order. The value of n is equal to the highest order of these functions and can attain the values of $-1, 1, 3, 5$ ... It can be assumed that the system of equations (*8*) will have a single solution just when all coefficients a_{ij} will be equal to zero. Then, the coefficients $a_{1(n+3)/2}$, $a_{2(n+3)/2}$, and $a_{3(n+3)/2}$ can serve us to calculate the functions $f_n(z)$, $g_n(z)$, and $h_{n-1}(z)$ (functions of n-th order). If n is higher than -1, we can derive recurrent formulae for the functions of the n-th order calculated from the known functions of the $n - 2$, $n - 4$, and lower orders. After rearrangement, we obtain a system of differential equations

$$\sum_i \sum_j [-jf_i'f_j' + (j - 1) f_i''f_j - g_ig_j] - (n - 3)(n - 1) f_{n-2}' = \\ = 2g_n + f_n''' + (n - 1) h_{n-1}, \quad (9)$$

$$\sum_i \sum_j [(j - 1) f_i'g_j - (i - 1) f_ig_j'] + (n - 3)(n - 1) g_{n-2} = 2f_n' - g_n'', \quad (10)$$

$$\sum_i \sum_j [(i - 1)(i - j + 2) f_if_j'] + (n - 3) f_{n-2}'' + (n - 5)(n - 3)^2 f_{n-4} = h_{n-1}'. \quad (11)$$

Here, the summation indices acquire values beginning from 1 and satisfying the condition $i + j + 1 = n$ for Eqs (*9*) and (*10*) and $i + j + 3 = n$ for Eq. (*11*); they are always odd.

On differentiating Eq. (*9*) twice with respect to z and using Eqs (*10*) and (*11*) we eliminate the functions g_n and h_{n-1}; after rearrangement we arrive at the following differential equation of the fifth order:

$$f_n^{(V)} + 4f_n' = \sum_{\substack{i \; j \\ (i+j+1=n)}} \sum [-jf_i'f_j''' - (j + 1) f_i''f_j'' + (j - 2) f_i'''f_j' + (j - 1) f_i^{(IV)}f_j - \\ - g_i''g_j - 2g_i'g_j' - g_ig_j'' + 2(j - 1) f_i'g_j - 2(i - 1) f_ig_j'] - (n - 1) . \\ . \sum_{\substack{i \; j \\ (i+j+3=n)}} \sum [(i - 1)(i - j + 2)(f_i'f_j' + f_if_j'')] + (n - 1)(n - 3) [2g_{n-2} - 2f_{n-2}''' - \\ - (n - 3)(n - 5) f_{n-4}'] . \quad (12)$$

This can be put into the simple form

$$f_n^{(V)} + 4f_n' = R_n, \quad (13)$$

where R_n denotes the right-hand side of Eq. (*12*). Hence it follows an important conclusion that the function f of any order is defined formally by the same differential equation (only its right-hand side varies). For an analytical solution of this problem, it is sufficient to find a particular integral satisfying the right-hand side, since the solution of the corresponding homogeneous differential equation can easily be found. Eq. (*13*) also suggests that the functions f, g, and h can be calculated numerically (the higher-order functions can be determined if necessary).

A solution for the functions of at most the third order[8] can be shortened and modified to take the form

$$f'_{-1} = 0\,, \quad f'_1 = A_1 \cosh z \cos z + B_1 \sinh z \sin z\,,$$

$$f_3 = A_3 \cosh z \sin z + B_3 \sinh z \cos z - (A_1^2 + B_1^2)(\sinh 2z - \sin 2z)/20 + (h/8)\, z f'_1(z)\,,$$

$$f'_3 = (A_2 + B_3) \cosh z \cos z + (A_3 - B_3) \sinh z \sin z - (A_1^2 + B_1^2)(\cosh 2z - \cos 2z)/10 + (h/8)(z f''_1(z) + f'_1(z)]\,,$$

$$g_{-1} = 0\,, \quad g_1 = -\tfrac{1}{2}[f'''_1(z) - f'''_1(\alpha)]\,,$$

$$g_3 = -\tfrac{1}{2}[f'''_3(z) - f'''_3(\alpha) + (f'_1(z))^2 + (g_1(z))^2]\,,$$

$$h_{-2} = 0{\cdot}5\,, \quad h = f'''_1(\alpha)\,, \quad h_2 = -0{\cdot}5 f'''_3(\alpha)\,.$$

The constants A_n and B_n are determined by the boundary conditions

$$f_1(\alpha) = \alpha Re\,, \quad f'_1(\alpha) = 0\,.$$

$$f_3(\alpha) = 0\,, \quad f'_3(\alpha) = 0\,.$$

Our task is to find the functions of the fifth order, whereby the accuracy of the calculation should be improved. Therefore, we have to solve Eq. (*13*) with the right-hand side

$$R_5 = (-4f'_1 f'_3 + 2f_3 f''_1 - 2g_1 g_3 - 16 f'_2)'' + 4g_3 f'_1 + 16 g_3 - 4 f_3 g'_1\,. \quad (14)$$

Again, we assume that the function $f_5(z)$ is given by the sum of the solution of the corresponding homogeneous differential equation and particular integral $P_5(z)$:

$$f_5 = A_5 \cosh z \sin z + B_5 \sinh z \cos z + P_5(z)\,. \quad (15)$$

Accordingly,

$$f'_5 = (A_5 + B_5) \cosh z \cos z + (A_5 - B_5) \sinh z \sin z + P'_5(z)\,. \quad (16)$$

The problem consists in finding the particular integral. The right-hand side of Eq. (*14*) is rearranged and after analysing the properties of the functions of the fifth order, the first derivative of the particular integral is found in the form (resembling the modified expression R_5)

$$\begin{aligned} P_5' = {} & K_1[2z(\cosh 2z - \cos 2z)]' + K_2 \cosh 2z + K_3 \cos 2z + \\ & + K_4[2(\cosh 2z - \cos 2z)]'' + K_5 \sinh 2z \sin 2z + K_6 \cosh 2z \cos 2z + \\ & + K_7(z \cosh z \cos z)' + K_8(z \sinh z \sin z)' + K_9 \sinh 3z \sin z + \\ & + K_{10} \sinh z \sin 3z + K_{11} \cosh 3z \cos z + K_{12} \cosh z \cos 3z + K_{13}(z^2 f_1)' + \\ & + K_{14}(z f_1''')' + K_{15}(z^2 f_1'')' + K_{16}(z f_1')' + K_{17} \,. \end{aligned} \tag{17}$$

The constants K_1 through K_{17} are defined as follows:

$$\begin{aligned} K_1 &= -(A_1^2 + B_1^2)\, h/160 \equiv -Q_0 h/8 \,, \quad Q_0 = (A_1^2 + B_1^2)/20 \,, \\ K_2 &= \tfrac{1}{10}(-5A_1A_3 - 2B_1A_3 - 2A_1B_3 + 5B_1B_3 + 56Q_0) \,, \\ K_3 &= \tfrac{1}{10}(2A_1A_3 + 5B_1A_3 + 5A_1A_3 - 2B_1B_3 + 56Q_0) \,, \\ K_4 &= Q_0 h/4 \,, \quad K_5 = 11h(B_1^2 - A_1^2)/480 \,, \quad K_6 = 11hA_1B_1/240 \,, \\ K_7 &= \tfrac{1}{8}[4Q_0A_1 - 4hB_1 + 24(A_3 - B_3) + h(A_3 + B_3)] \,, \\ K_8 &= \tfrac{1}{8}[4hA_1 - 4Q_0B_1 - 24(A_3 + B_3) + h(A_3 - B_3)] \,, \\ K_9 &= Q_1(B_1 + 3A_1) \,, \quad K_{10} = Q_1(B_1 - 3A_1) \,, \quad K_{11} = Q_1(A_1 - 3B_1) \,, \\ K_{12} &= Q_1(A_1 + 3B_1) \,, \quad Q_1 = 9Q_0/80 = 9(A_1^2 + B_1^2)/1\,600 \,, \\ K_{13} &= -3h/8 \,, \quad K_{14} = 3h/32 \,, \quad K_{15} = h^2/128 \,, \quad K_{16} = -3h^2/128 \,, \\ K_{17} &= \tfrac{1}{2}(\tfrac{1}{8}hA_1B_1 - h^2 - 8h_2 - A_1A_3 - B_1A_3 + A_1B_3 - B_1B_3) \,. \end{aligned}$$

The constants A_5 and B_5 are found from the boundary conditions

$$f_5(\alpha) = 0 \,, \quad f_5'(\alpha) = 0 \,.$$

The functions g_5 and h_4 are found from Eqs (*10*) and (*11*):

$$g_5 = \tfrac{1}{2}(-4f_1'f_3' + 2f_3f_1'' - 2g_1g_3 - 16f_3' - f_5''' + f_5'''(\alpha)) \,,$$
$$h_4 = 2f_3' - \tfrac{1}{4}f_5'''(\alpha) \,.$$

A program for the numerical calculations was written in FORTRAN and the solution was carried out on an ICL 4-72 type computer for $\alpha = 7{\cdot}0$ and several values of the *Re* number. The velocity field was obtained in the whole cross section between the disc electrodes up to a distance $\bar{r}/a = 25$.

RESULTS

The results obtained are in support of the assumption that the functions of the fifth and higher orders influence the velocity field only for $\bar{r}/a \leqq 25$, *i.e.* for the inlet region, and for higher values of α and *Re*. *E.g.*, for $\alpha = 7$ and $Re = 41$ the fifth-order functions have no influence on the calculated radial velocity u even for $\bar{r}/a = 10$.

A comparison of the previous[8] and present calculations for $Re = 1\,243$ and $\bar{r}/a = 18$ (Fig. 2) shows that the dimensionless radial velocity is roughly the same. A small difference consists in that close to the central plane of symmetry ($\bar{z}/a = 0$) the radial velocity attains negative values, *i.e.* the liquid flows towards the inlet orifice. The point at which $u = 0$ (where u changes its sign) is somewhat shifted towards the central plane (from $\bar{z}/a = 0{\cdot}5$ to $0{\cdot}45$), and the maximum value of u is by about 12% lower. For $\bar{r}/a = 25$, the velocity fields are practically the same.

A comparison of the previous[8] and present calculations for $Re = 2\,072$ and $\bar{r}/a = 18$ and 25 (Figs 3 and 4) again indicates the possible existence of a back flow close to the central plane, which becomes apparently more pronounced when proceeding towards the inlet of the liquid. The point of zero radial velocity is again somewhat shifted towards the central plane, namely to 0·4 and 0·45 for $\bar{r}/a = 18$ and 25, respectively. The maximum value of u is by about 10% lower. For lower values of $\bar{r}/a$, the term with the function $f_5'(z)$ becomes important for the radial velocity field. Since this function shows large oscillations close to the disc (for $\bar{z}/a$ approaching α), the calculation gives in this region strong back flows, which does not correspond to reality. A similar behaviour was observed in the case of the calculation involving third-order functions, although to a lesser extent. The values of u for various values of *Re* are illustrated in Fig. 5, showing a pronounced influence of the *Re* number.

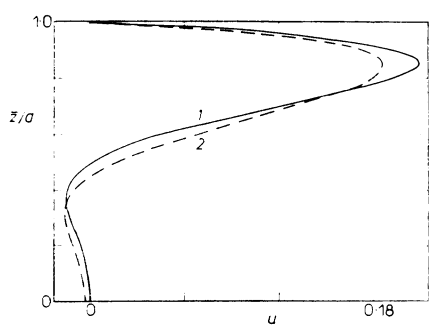

FIG. 2

Comparison of the calculated radial velocity for $\alpha = 7$, $Re = 1\,243$, $\bar{r}/a = 18$. *1* Calculation using functions of the third order[8], *2* calculation using functions of the fifth order

The course of the normal velocity w is shown in Figs 6 and 7. There are two differences against the previous calculation[8]: for higher Re values, the absolute values of w are about twice as large, which is however unimportant since the highest values of w are much lower than those of the radial and tangential velocities and they decrease rapidly with increasing r. Secondly, the point of $w = 0$ is shifted from $\bar{z}/a = 0{\cdot}6$ to $0{\cdot}53$, which partly corresponds to the shift of the point of $u = 0$. Hence, mixing of the liquid at the discs with that at the central plane should not take place. The maxima and minima of the curves are shifted only very little towards the central plane.

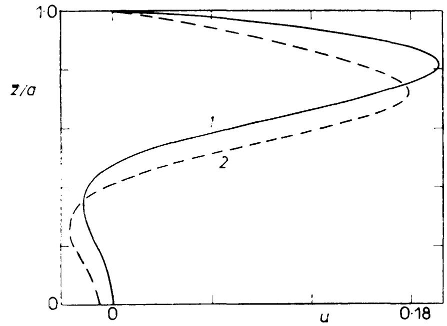

FIG. 3

Comparison of the calculated radial velocity for $\alpha = 7$, $Re = 2\,072$, $\bar{r}/a = 18$. *1* Calculation using functions of the third order[8], *2* calculation using functions of the fifth order

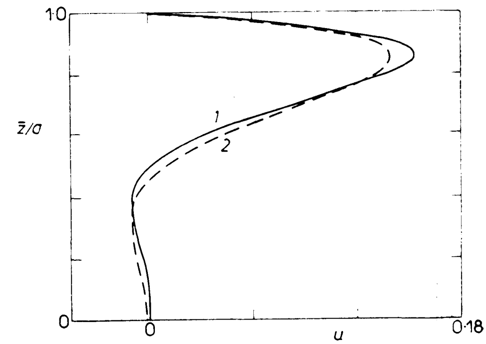

FIG. 4

Comparison of the calculated radial velocity for $\alpha = 7$, $Re = 2\,072$, $\bar{r}/a = 25$. *1* Calculation using functions of the third order[8], *2* calculation using functions of the fifth order

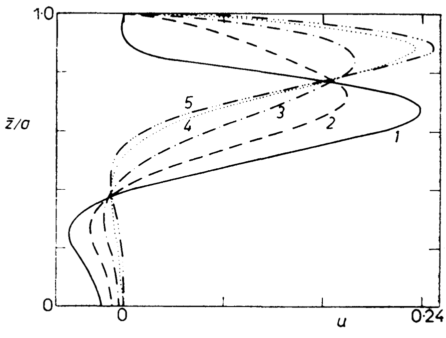

FIG. 5

Radial velocity calculated for $\alpha = 7$, $\bar{r}/a = 18$ by using fifth-order functions. Values of Re: *1* 2 900; *2* 2 072; *3* 1 243; *4* 414; *5* 41

The course of the dimensionless tangential velocity v in a cross-section between the discs is shown in Figs 8 and 9 (the ratio of v/v_{max} is plotted, where v_{max} is the maximum value of v on the disc surface). These curves are almost identical to the preceding ones[8] The difference is that at higher *Re* values the absolute values are higher. The back tangential flow for $Re = 2\,900$ represents more than 90% of the distance between the discs. The points of the minima on the curves have not changed appreciably.

Whereas the velocity fields show no essential changes, the static pressure is different (Fig. 10). For low *Re* values and large distances from the inlet, the difference of $p - p_0$ changes only little (p_0 is the static pressure at $\bar{r}/a = 25$), however at lower distances $\bar{r}/a$ the newly calculated term in the expansion (7) becomes important (its value is positive for $\alpha = 7$). Therefore, all curves are directed towards infinity. It may be assumed that a further term could change this situation, however the values of $p - p_0$ beginning from a certain distance $\bar{r}/a$ are sufficiently accurate regardless of further terms in the expansion (7). Curves *1* and *3* in Fig. 10 increase slowly from $\bar{r}/a = 13$ corresponding to sucking of the liquid between the rotating discs.

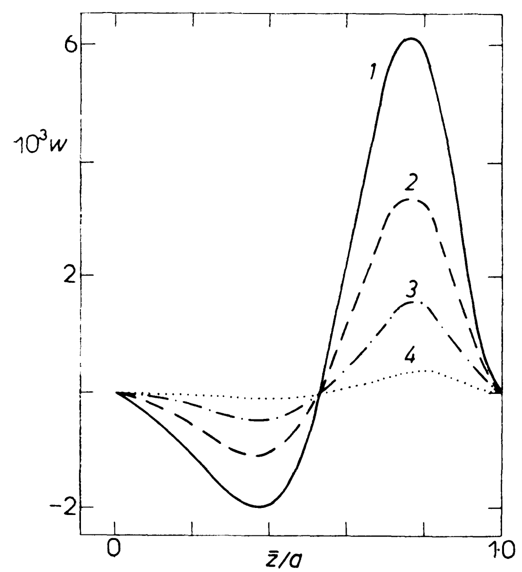

FIG. 6

Normal velocity calculated for $\alpha = 7$, $\bar{r}/a = 18$ by using fifth-order functions. Values of *Re*: *1* 2 900; *2* 2 072; *3* 1 243; *4* 414

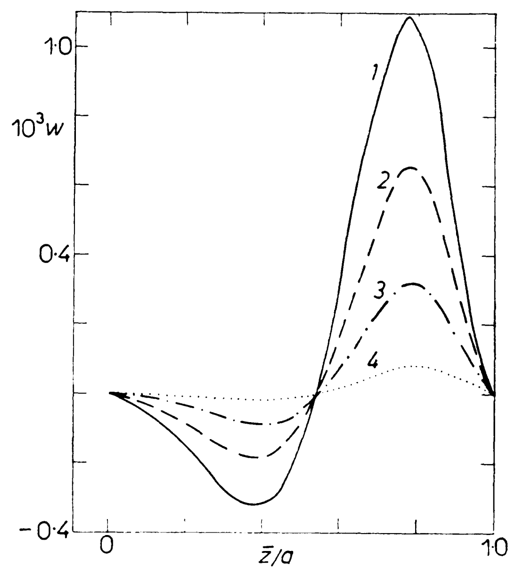

FIG. 7

Normal velocity calculated for $\alpha = 7$, $\bar{r}/a = 25$ by using fifth-order functions. Values of *Re* as in Fig. 6

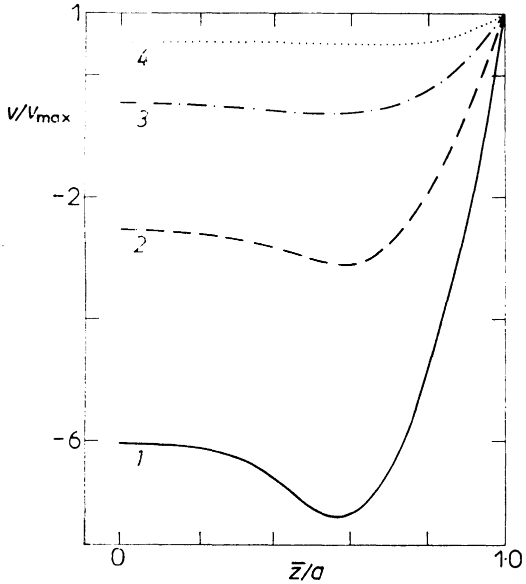

FIG. 8

Tangential velocity (v/v_{max}) calculated for $\alpha = 7$, $\bar{r}/a = 25$ by using fifth-order functions. Values of *Re* as in Fig. 6

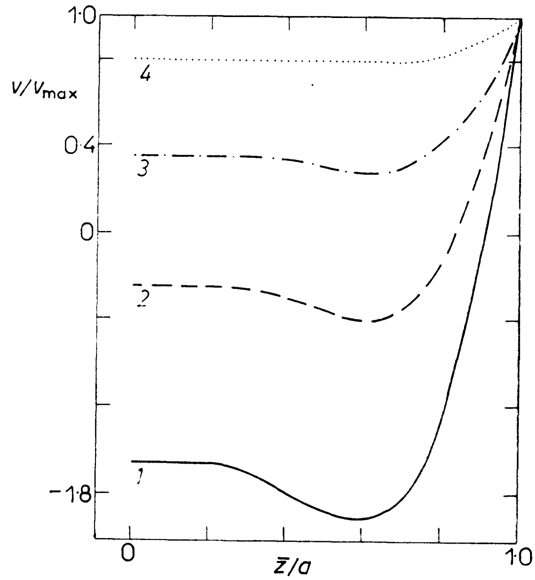

FIG. 9

Tangential velocity (v/v_{max}) calculated for $\alpha = 7$, $\bar{r}/a = 25$ by using fifth-order functions. Values of *Re* as in Fig. 6

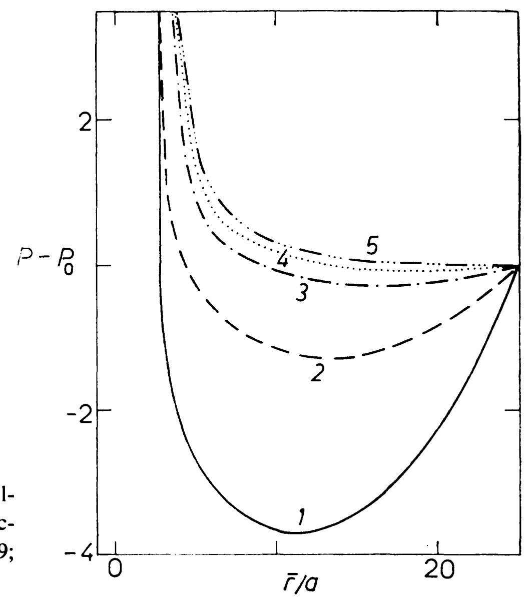

FIG. 10

Dimensionless static pressure ($p - p_0$) calculated for $\alpha = 7$ by using fifth-order functions. Values of *Re*: *1* 290; *2* 414; *3* 622; *4* 829; *5* 1 243

For $Re = 829$, curve *4* decreases to zero from $\bar{r}/a = 13$ to 25, and curve *5* for $Re = 1\,243$ is decreasing. It can hence be estimated that for $\alpha = 7$ and zero hydrodynamic resistance in the inlet the rate of flow between the discs will correspond to $Re = 800-900$.

DISCUSSION

A comparison of the results obtained with functions of the fifth and third order[8] for the Taylor criterion $\alpha = 7$ shows that the functions of the fifth order used in the present work do not lead to principally different velocity fields. They play almost no role in the region $\bar{r}/a \leqq 25$ (beyond the inlet region), while for lower values of $\bar{r}/a$ they lead to some changes of absolute values and characteristic points. The static pressure is markedly influenced, the newly calculated term in the expansion (7) being positive so that all curves in Fig. 10 increase towards $+\infty$. The correctness of the calculation cannot be judged since a comparison with experiments is lacking. It can be assumed that the limiting values of $\bar{r}/a$ given[8] as 25 shifted somewhat towards the inlet region, *i.e.* to the centre of the discs. Although a calculation of functions of a still higher order appears possible, no significant improvement of the theory can be expected.

LIST OF SYMBOLS

a	half-distance of the discs
$\bar{p}$	static pressure
p	dimensionless static pressure ($\bar{p}/\varrho v_1^2$)
Q	volume rate of flow
$\bar{r}$	radius
r	dimensionless radius ($\alpha\bar{r}/a$)
Re	Reynolds number (av_1/ν)
$\bar{u}$	radial velocity
u	dimensionless radial velocity ($\bar{u}/v_1$)
$\bar{v}$	tangential velocity
v	dimensionless tangential velocity ($\bar{v}/v_2$)
v_1, v_2	parameters equal to $Q/4\pi a^2$ and $(\omega\nu)^{1/2}$, respectively
$\bar{w}$	normal velocity
w	dimensionless normal velocity ($\bar{w}/v_2$)
$\bar{z}$	distance from central plane
z	dimensionless distance from central plane ($\alpha\bar{z}/a$)
α	Taylor number, equal to $a(\omega/\nu)^{1/2}$
ν	kinematic viscosity
ω	angular velocity

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